Quiz 7 summary

#2) Need to pick C so that

$$\int_0^2 \int_0^1 \frac{Cx(1+x)}{6}\, dx\, dy = 1$$

#1) $$\int_0^{2\pi} \int_0^{\pi/6} \int_0^{7\sec\phi} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$0 \le \theta \le 2\pi$ (covers all possible values of θ)

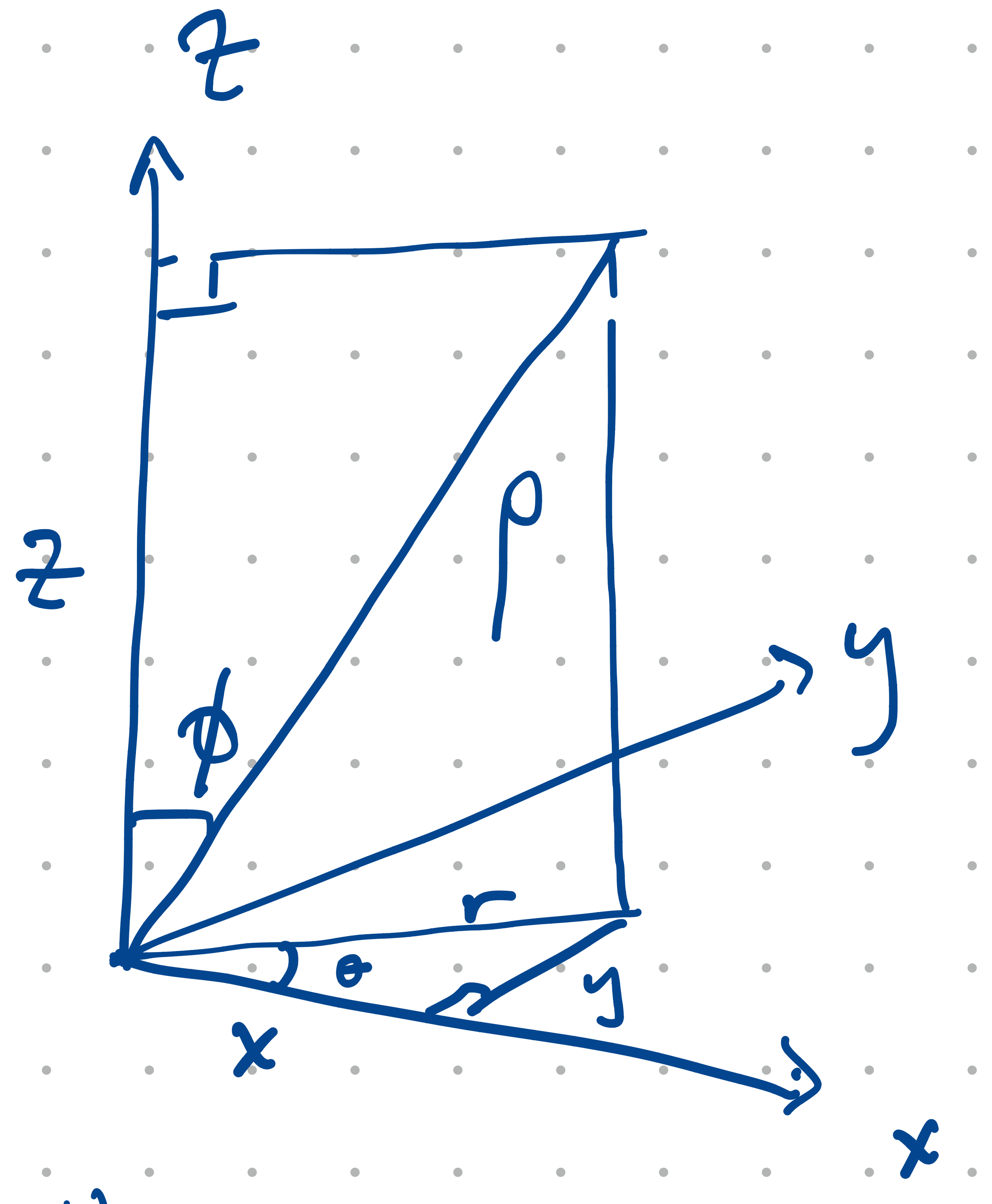
$0 \le \phi \le \pi/6$

$0 \le \rho \le 7\sec\phi$

note: $\tan\phi = r/z = \sqrt{x^2+y^2}/z$

so $$z \ge \tan\left(\frac{\pi}{6}\right)\sqrt{x^2+y^2}$$

$\underbrace{\rho\cos\phi}_{z} \le 7$



So it's a (solid) cone w/ a flat top.

#3) "Above the surface $z = x^2+y^2$"

means $z \ge x^2+y^2$

$\left(\begin{array}{c} r \ge 0 \\ 0 \le \theta \le 2\pi \end{array} \right)$

"below the plane $z=2y$"

means $z \le 2y$.

$r^2 \le z \le 2r\sin\theta$ (z-bounds)

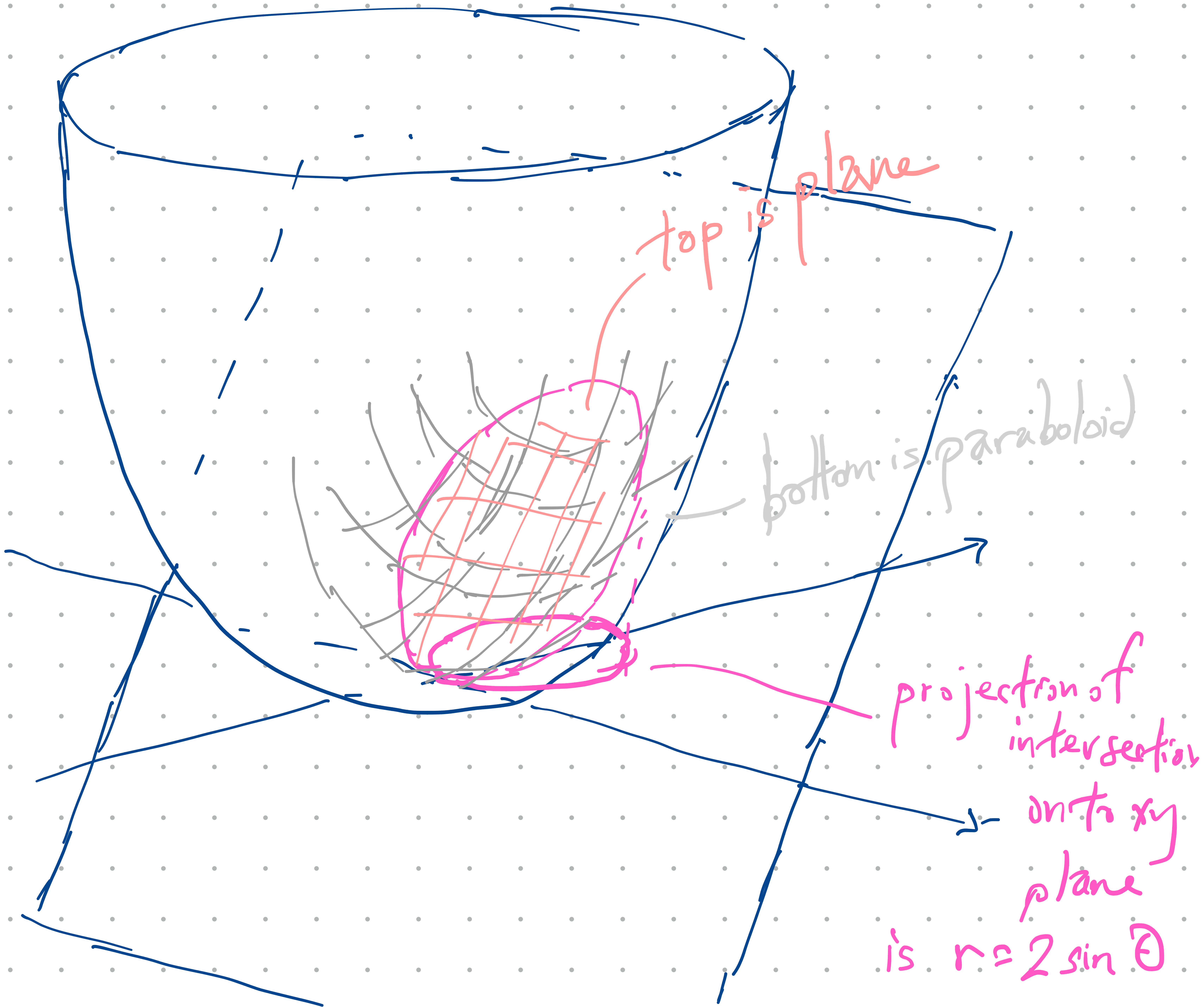
For r bounds:

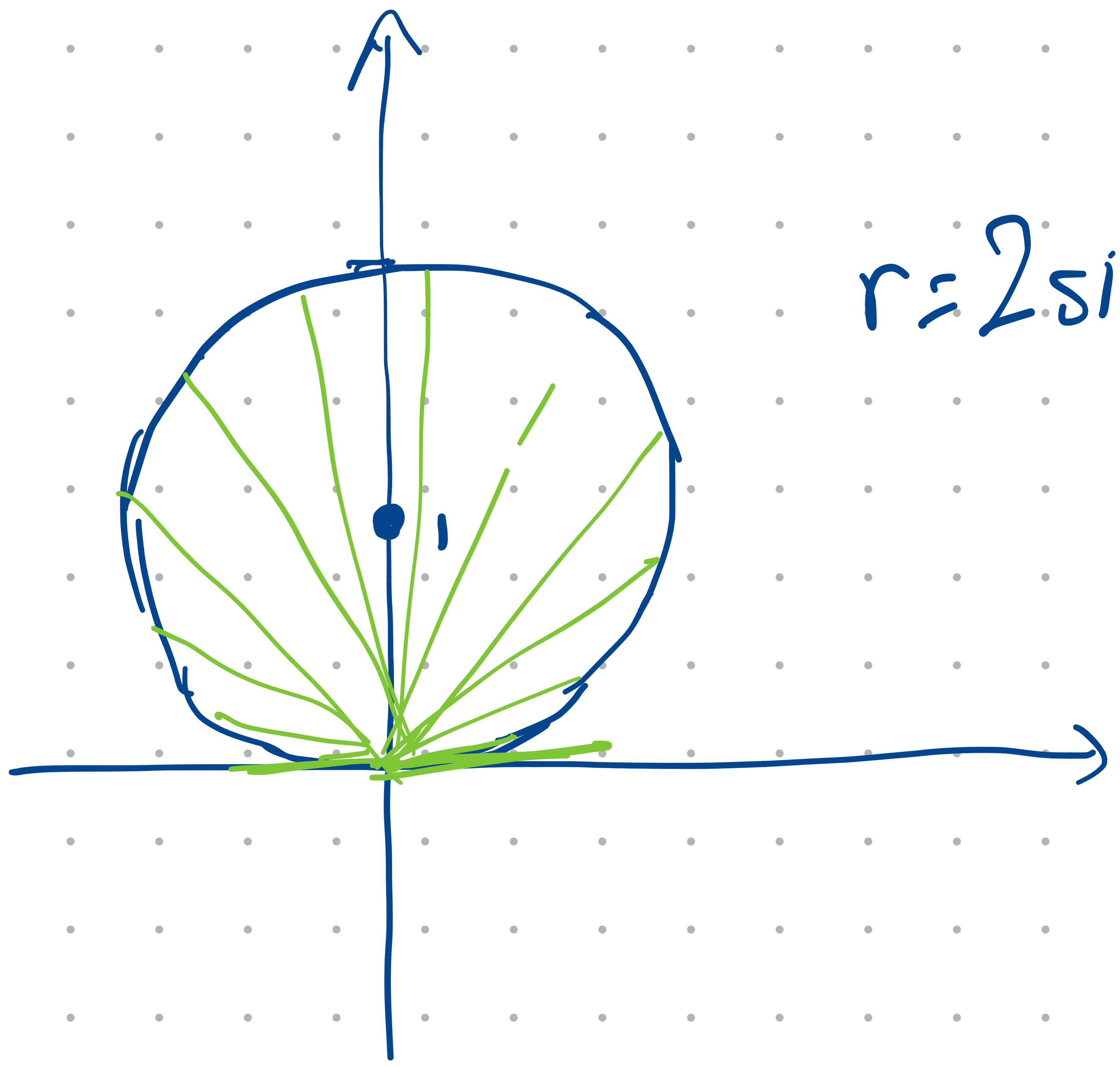
$$r^2 \leq 2r\sin\theta$$

$$0 \leq r \leq 2\sin\theta \quad \text{(r-bounds)}$$

For θ bounds:

$$0 \leq 2\sin\theta \quad \text{i.e.} \quad 0 \leq \theta \leq \pi \quad (\theta\text{-bounds})$$





$r = 2\sin\theta$ is this circle

(recall:

$$r^2 = 2r\sin\theta$$

$$x^2 + y^2 = 2y$$

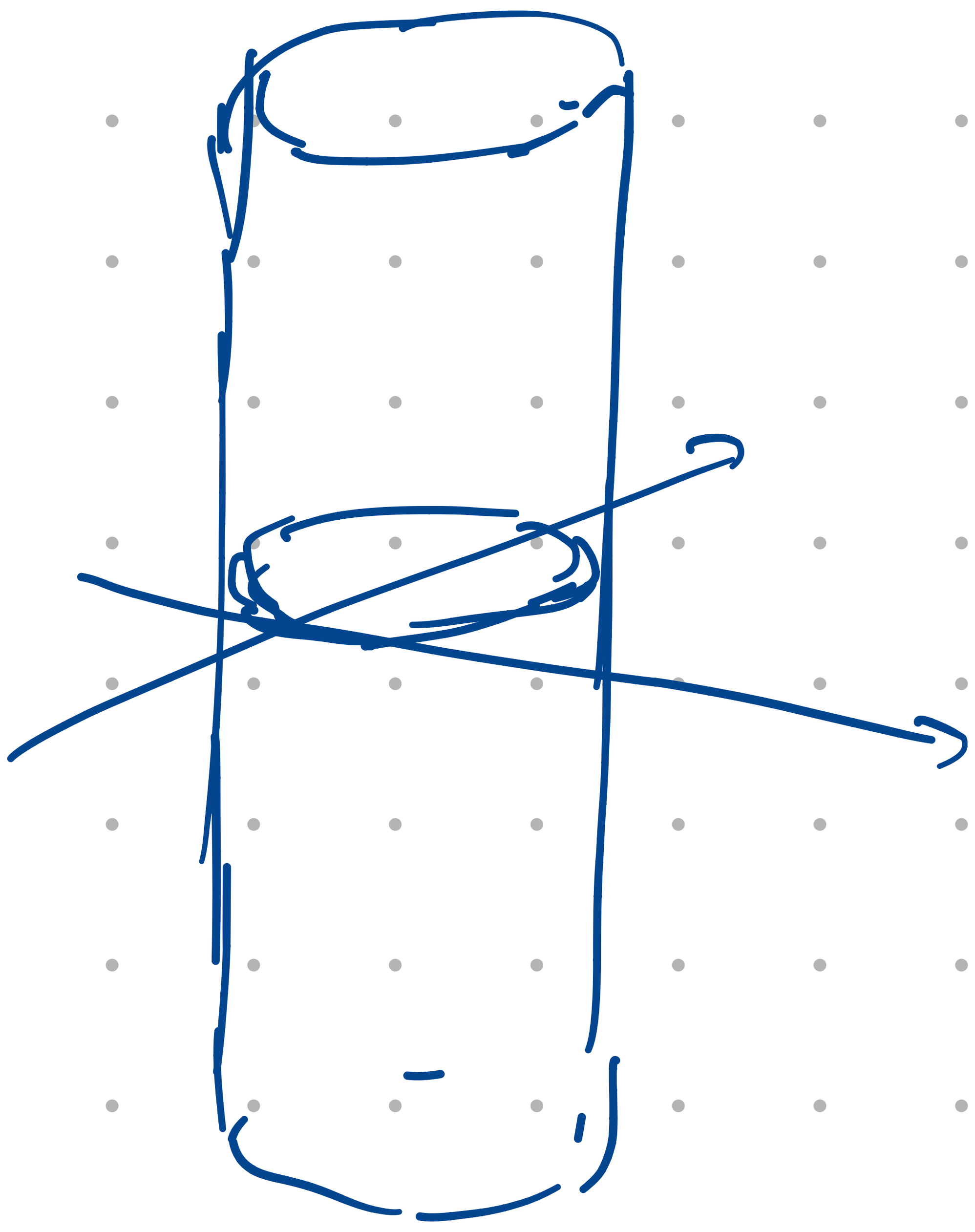
$$x^2 + y^2 - 2y + 1 = 1$$

$$x^2 + (y-1)^2 = 1$$)

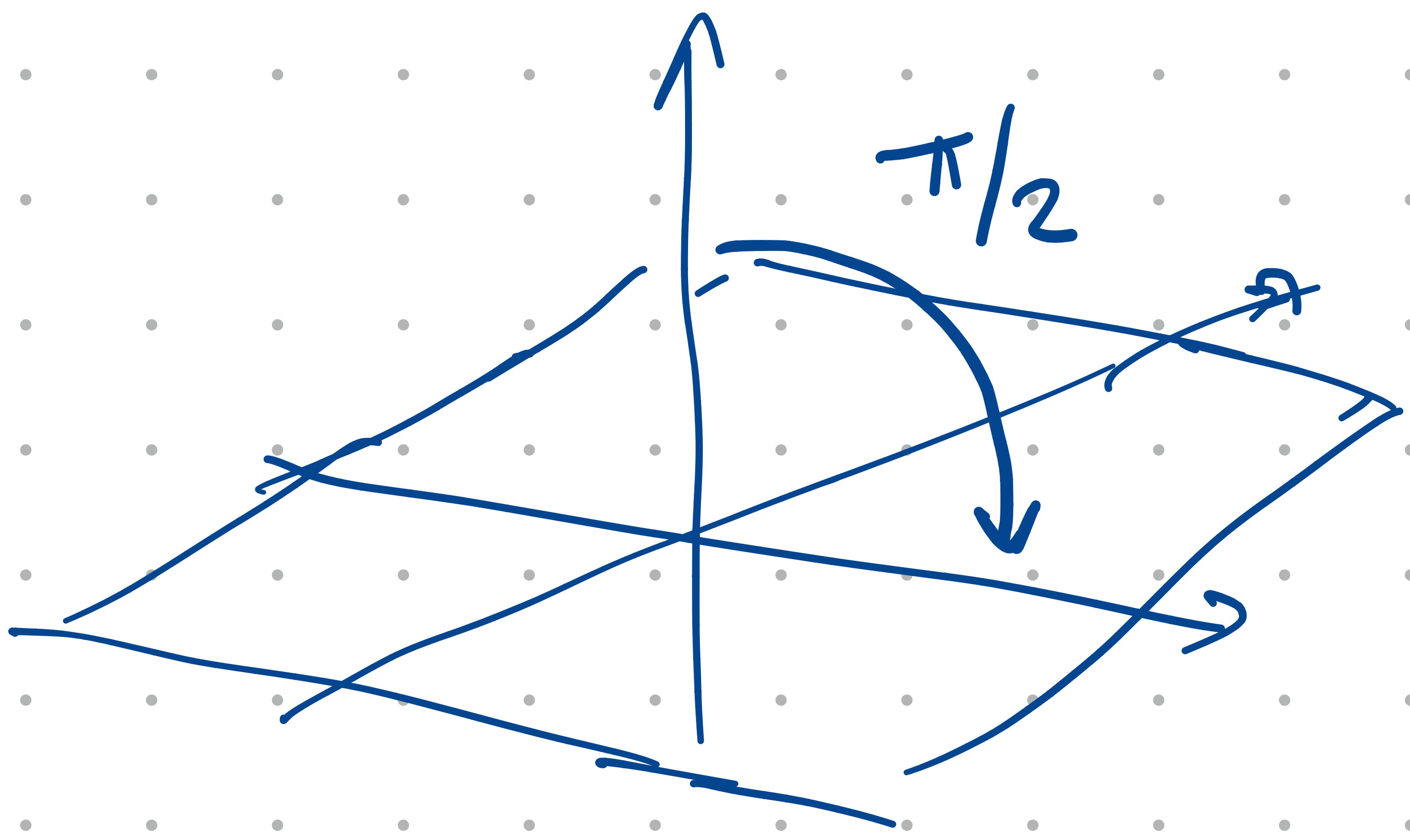
$$\int_0^{\pi} \int_0^{2\sin\theta} \int_{r^2}^{2r\sin\theta} 1 \, r\,dz\,dr\,d\theta$$

1) In polar (i.e. r, θ), $r = \sin\theta$ describes a circle

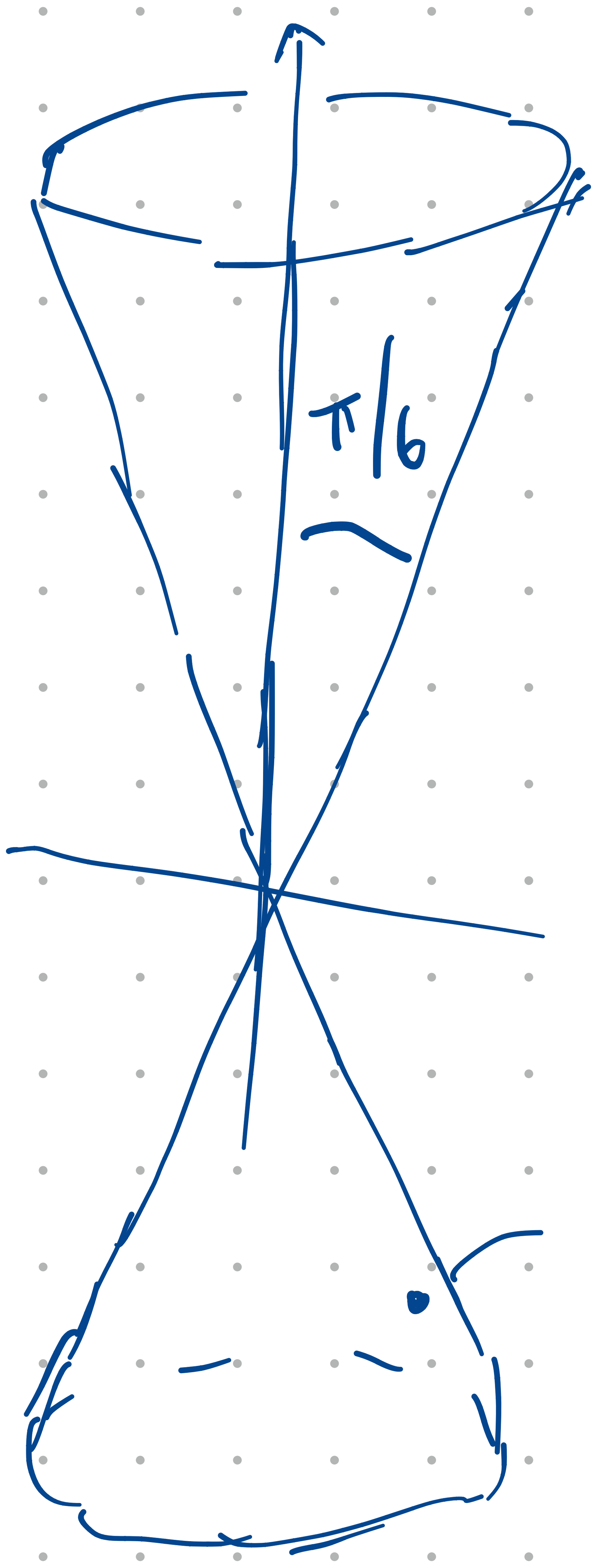
In cylindrical (i.e. r, θ, z) $r = \sin\theta$ describes a cylinder.



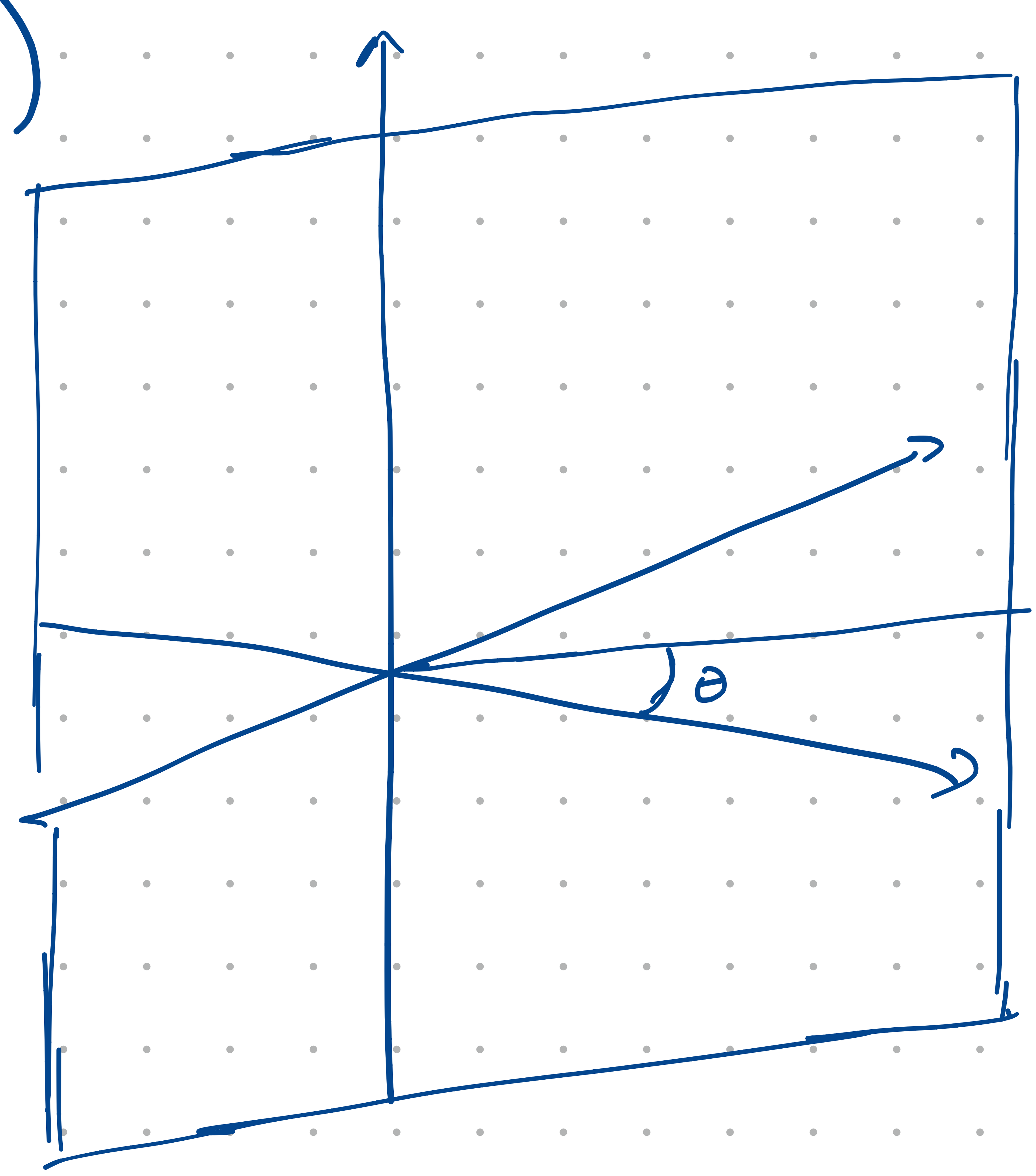
3) $\phi = \pi/2$ is just the xy-plane!



2)



$\phi = \pi/6$ but ρ is negative.

4) 

plane.

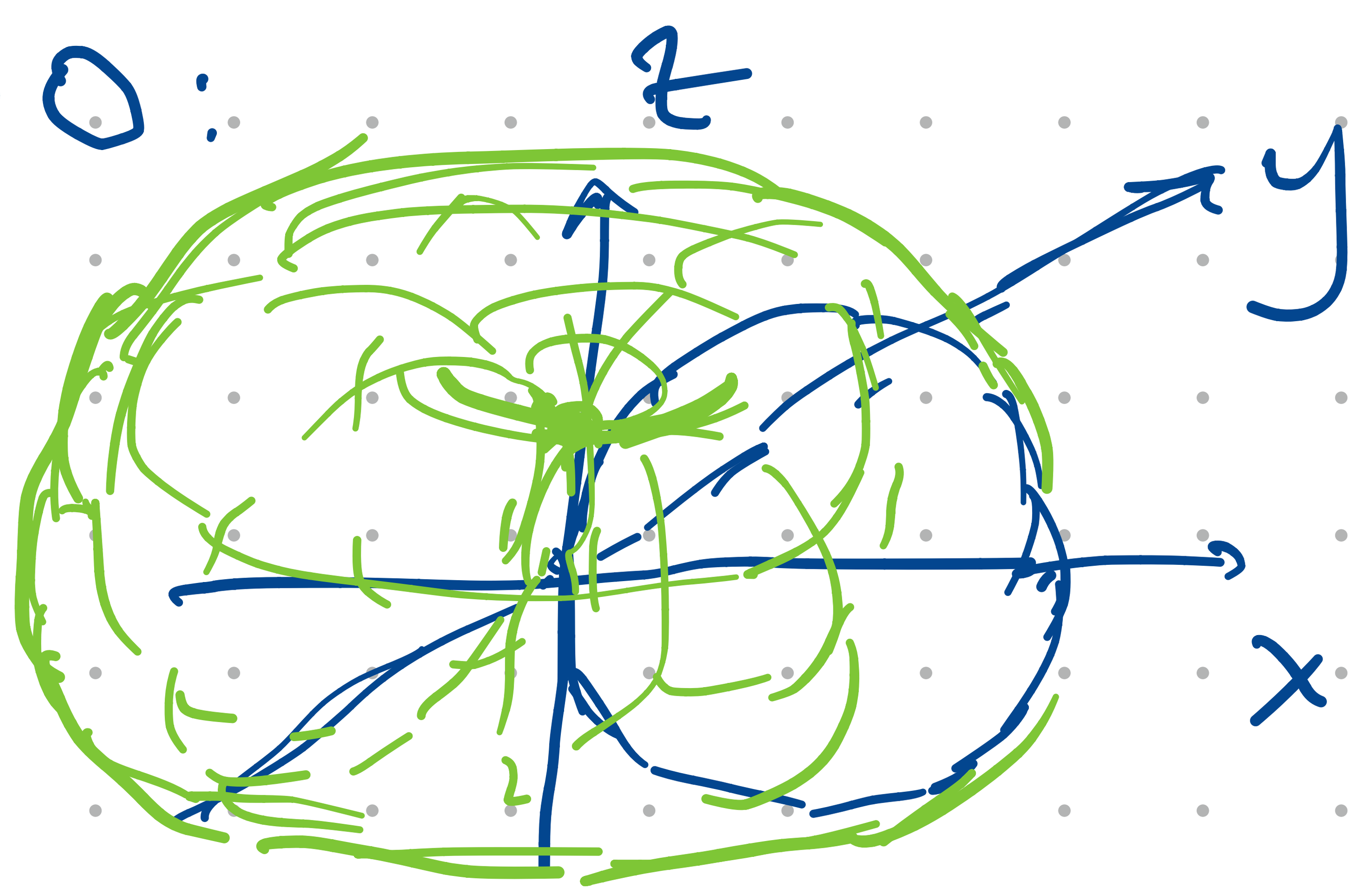
5) $\rho = \cos\phi$

$\rho^2 = \rho\cos\phi$

$x^2+y^2+z^2 = z$ complete the square.

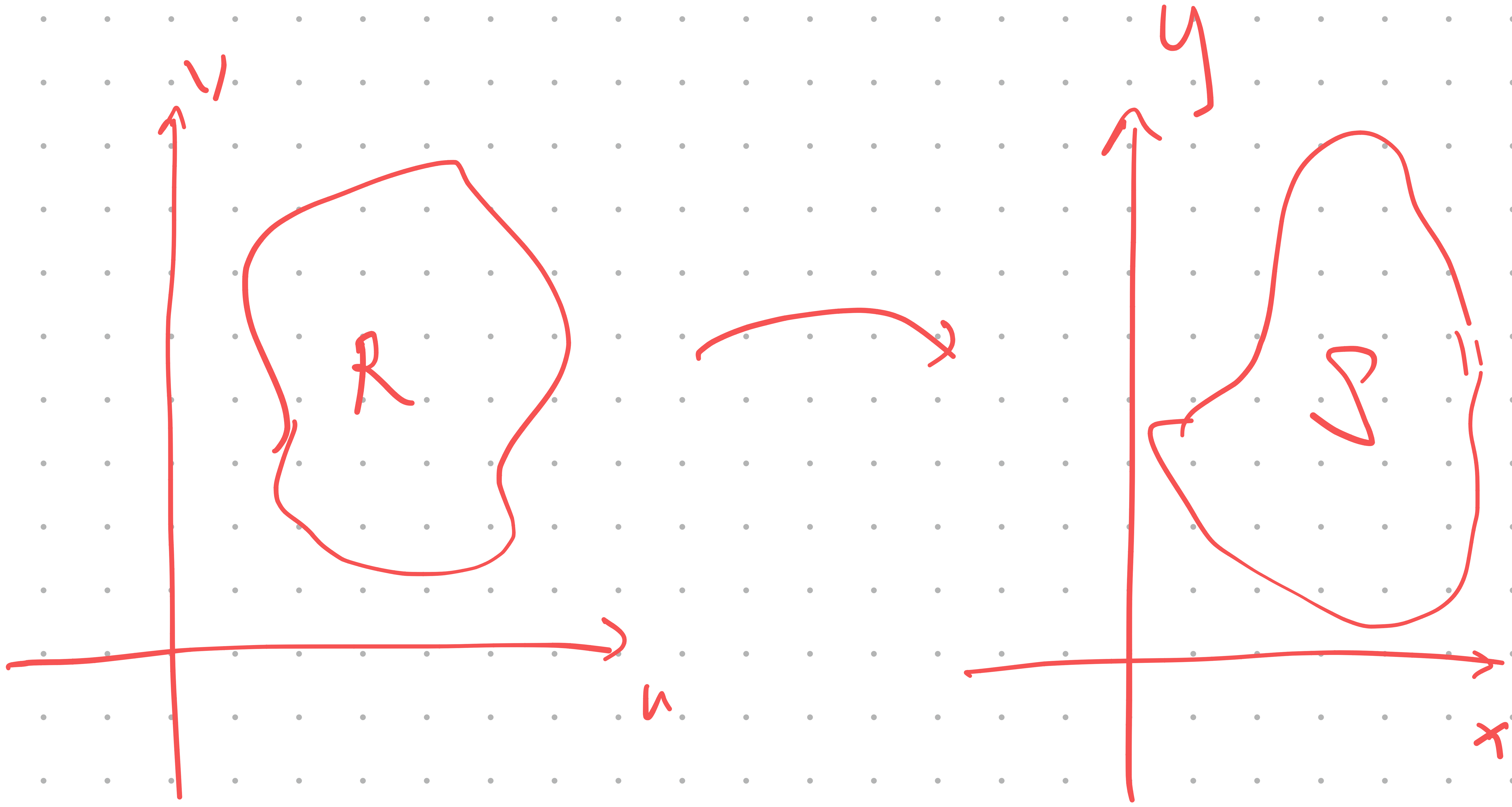
to find it's a sphere.

6) $\rho = \sin\phi$ when $\theta = 0$:



indep. of θ:

For 7-8: when doing change of variables, there should be exactly one solution in R for (u,v) for each (x,y) in S



#7 is not fixable.

#8 is fixable: just shrink the u,v region to only be the right half.